The current polarization rectification of the integer quantized Hall effect

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Abstract

We report on our theoretical investigation considering the widths of quantized Hall plateaus (QHPs) depending on the density asymmetry induced by the large current within the out-of-linear response regime. We solve the Schrödinger equation within the Hartree type mean field approximation using Thomas Fermi Poisson nonlinear screening theory. We observe that the two dimensional electron system splits into compressible and incompressible regions for certain magnetic field intervals, where the Hall resistance is quantized and the longitudinal resistance vanishes, if an external current is imposed. We found that the strong current imposed, induces an asymmetry on the IS width depending linearly on the current intensity.

Key words: Edge states, Quantum Hall effect, Out of linear response, Rectification
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1. Introduction

The phenomenon of the integer quantized Hall effect (IQHE) [1] continuous to hold interest as newer and newer types of hetero-junctions [2,3,4] are produced. The early attempts to explain the IQHE, like the bulk [5] or the edge [6,7] pictures, considers electron-electron interactions to be irrelevant and attributes the effect either to disorder or to the bending of the confinement potentials, respectively. From theses theories, it is known that the widths of the QHP depend on the electron density, mobility, temperature and the amplitude of the applied current. However, the direction of the applied current is not considered to be influencing the plateau widths. However, the inclusion of the (direct) Coulomb interaction numerically [8,9] or analytically [10] enriches the physics beyond the single particle pictures. The utilization of the local Ohm's law [11] together with the self-consistent numerical calculations allowed Siddiki and Gerhardts to calculate the quantized Hall plateaus and also the transition between the plateaus [9], within the linear response regime. A further investigation considering the out of linear response regime showed that the widths of current carrying egde-states linearly depend on the current intensity based on the electron-electron interactions [12]. In this work, we obtain the widths of the QHPs from a model which is purely based on the electron-electron interactions, supported by the local Ohm's law [9]. We solve the Schrödinger and the Poisson equations

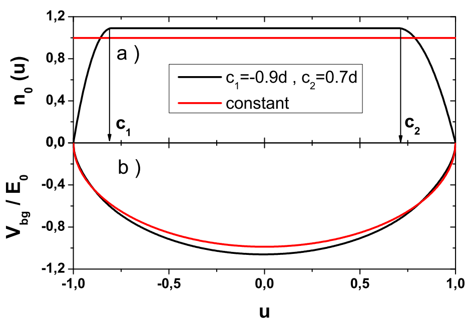

Fig. 1. The cross section of the donor layer considering (a) for two values of steepness parameters which are c_1 left side and c_2 right side parameters of donor distribution. The red line represents a constant donor distribution.

self-consistently within the Thomas-Fermi approximation [13], which implicitly assumes that the potential landscape varies slowly on the quantum mechanical length scales. We start from a homogeneous donor distribution to calculate the confinement potential (in fig. 1a depicted with red line), which we use as an initial condition for our iterative numerical technique. We then consider an inhomogeneous distribution of the donors to obtain different potential slopes at the two edges of the sample (in fig. 1a depicted with black line). Background potentials generated by the donor distributions are shown in fig. 1b with the same color code. As we show later, by doing so we directly change the widths of the incompressible strips (ISs) resulting from the screening. Within these regions the backscattering is suppressed, therefore current is confined at the ISs, hence any effect that influences the widths of the ISs will effect the current and potential distribution in the sample. It was shown that, if there exists an IS somewhere in the sample the system is in a QHP [14]. The self-consistent model, predicts that the widths of the ISs will also be modified by the imposed current, namely by the amplitude [15]. If a DC current is passed in the +y direction, due to the tilting of the Landau levels, the IS at the right hand side (RHS) enlarges, whereas, the IS on the left hand side (LHS) shrinks. Fig. 2a, depicts such a situation under current bias. Now if we start with a narrow IS on the LHS, it is possible to achieve equi-width ISs on both sides, by applying a certain imposed current, fig. 2b. As a result, we conclude that the widths of the QHPs also should depend on the applied current direction [15]. To summarize, by our self-consistent calculations we show that, the widths of the QHPs also depend on the current direction, which is in strong contrast to the conventional approaches.

The calculation scheme starts by determining the boundary conditions to describe the electronic system at hand: First, we assume a translation invariance in the current direction, *i.e.* $y-$, hence the electrostatic potential (therefore the y component of the electric field is also constant, E_y^0), second we consider a lateral confinement in x direction generated by a donor distribution $n_0(x)$ limited by top-side gates, which imposes the boundary conditions $V(-d) = V(d) = 0$, where d is the sample width. The analytical solution of the Poisson equation considering the above boundary conditions reads to the kernel

$$K(x, x') = \ln \left| \frac{\sqrt{(d^2 - x^2)(d^2 - x'^2)} + d^2 - x'x}{(x - x')d} \right|. \tag{1}$$

The confinement potential is obtained by the following integration for a given $n_0(x)$

$$V_{\rm bg}(x) = \frac{2e^2}{\bar{\kappa}} \int_{-d}^{+d} dx' n_0(x') K(x, x'), \tag{2}$$

where e is the electronic charge, $\bar{\kappa}$ an average dielectric constant and yields to

$$V_{\rm bg}(x) = -E_{\rm bg}^0 \sqrt{1 - (x/d)^2}, \quad E_{\rm bg}^0 = 2\pi e^2 n_0 d/\bar{\kappa}, \tag{3}$$

given that the donors are homogeneously distributed. However, as will be discussed later, we also consider an inhomogeneous donor distribution to create an asymmetric lateral confinement by considering a donor distribution described as below

$$n_0(x) = \left\{ \begin{array}{cc} -\frac{(u+c_1)^2}{(c_1-1)^2} + 1, & -1 \leq u < c_1 \\ 1, & c_1 \leq u < c_2 \\ -\frac{(u-c_1)^2}{(c_1-1)^2} + 1, & c_2 \leq u < 1 \end{array} \right\}.$$

By doing so we can controllably break the lateral confinement symmetry by setting c_1 and c_2

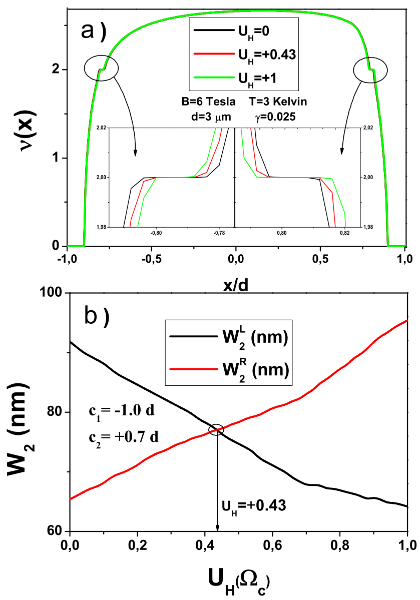

Fig. 2. The electron density as a function of lateral coordinate normalized with the sample width (a), for three selected current amplitudes ($U_{\rm H}$). Insets depict the regions, where incompressible strips reside. It is clearly seen that the IS at LHS becomes narrower by increasing UH, and opposite for the RHS. The widths of the ISs as a function of UH, when applying a positive current one can obtain equi-width ISs on both edges, regardless of the donor in homogeneity (b).

(almost) arbitrarily. Fig. 1a presents a situation considering a homogeneous donor distribution (i.e. $-c_1 = c_2 = 1$) and also a case where left side is more confining than the right side. Note that the donor number density is kept constant, that is the area below the the donor distribution curves are equal. The resulting confinement potentials are shown in fig. 1b, one can readily see that the asymmetric donor distribution leads a steeper bending on the left hand side (black line). The corresponding electron distribution in the absence of magnetic field B and vanishing temperature T is obtained from

$$n_{\rm el}(x) = D_0 \Theta(V_{bg}(x) - E_F), \tag{4}$$

where D_0 is a constant that corresponds to the two-dimensional density of states (DOS) in the absence of an applied B field and E_F is the Fermi energy fixed by the charge neutrality of the system. The next step is to calculate the interaction potential (energy) from

$$V_{\rm H}(x) = \frac{2e^2}{\bar{\kappa}} \int_{-d}^{+d} dx' n_{\rm el}(x') K(x, x'). \tag{5}$$

At finite temperatures the electron density is calculated from

$$n_{\rm el}(x) = \int dE D(E) f(E, \mu, kT, V(x)), \tag{6}$$

where $D(E)$ is the relevant DOS, $f(\epsilon)$ is the Fermi occupation function and μ is the electrochemical potential. Now by solving the total potential and the electron distribution iteratively, one can obtain the electrostatic quantities at equilibrium.

Once these quantities are known it is required to have a prescription which relates the electron density to the local conductivities [9] considering a fixed imposed current I, in our work we take this prescription from the self-consistent Born approximation [16]. At a first approximation one can neglect the effect of the imposed current on the electrostatic quantities (namely, the linear response) and the current distribution can be obtained simply by applying Ohm's law locally [11]. The Ohm's law states that the (local) potential drop is proportional to the local current times the local resistance (resistivity at 2D, with square normalization), i.e. we should look for drops at the self-consistently calculated potential. As an oversimplified picture, now we relate the screening properties of the electron gas in the presence of B field with the potential drop. Since the magnetic field Landau quantizes the system, there are two possibilities when considering the pinning of the Fermi energy to the Landau levels: 1) the E_F is equal to one of the Landau level, *the compressible state*, hence the DOS is high, and the system behaves like a metal. Therefore, as in all metals, the potential is constant and no current can flow with in these regions; 2) the E_F is not equal to the Landau energy the system is at the *incompressible state* and the self-consistent potential varies, hence the applied current flows from

these regions. In fig. 2a the calculated electron densities (in fact the filling factor, defined as $\nu(x) = 2\pi l^2 n_{\rm el}(x)$, with the magnetic length $l = \sqrt{eB/m}$) are shown considering an asymmetric donor distribution by setting $c_1 = -1$ and $c_2 = 0.7$. We see that the ISs are formed at both sides where the potential drops and density is constant considering three characteristic current biasses, U_H, which is measured in units of cyclotron energy (Ω_c). The ISs are highlighted at the insets, we see that at higher current densities the left ISs starts to shrink, whereas the right ISs becomes wider. The IS width dependency on the current amplitude is shown in fig. 2b. It is seen that the donor distribution asymmetry induced large IS at the left side (red line) starts to shrink when increasing the bias and its width becomes equal to the width of the right IS (black line) at $U_H = 0.43$. The effect of large bias current (out of linear response) implies that the formation of ISs strongly depends on the current amplitude, hence the QHPs also depends on the the polarization of the current. This can be seen by considering the slope of the Hall potential, say if the DC current is positive the Hall potential has an positive slope or vice versa. Now consider a potential drop at the IS which has an positive slope, the Hall potential will enlarge the IS on the right hand side. In the opposite situation the left IS is enhanced. Therefore depending on the current polarization one of the ISs will become leaky at a lower B, hence the quantized Hall effect is smeared [15]. A detailed investigation of the current polarization on the quantized Hall plateaus is discussed at Ref.[15].

2. Conclusion

For the high mobility, narrow and asymmetric samples we predict that, the large current either enlarges or shrinks the QHPs depending on whether the asymmetry induced by the current and the asymmetry caused by the edge profile coincides or not. Based on our findings, we proposed a sample structure where the effect of the current induced asymmetry and thereby the rectification of the QHPs can be controllably measured. As a final remark, we note that at the edge IQHE regime, due to the competition between the enhancement of the ISs resulting from the large current and suppression due to steep potential profile, therefore we expect a hysteresis like behavior in this regime both depending on the sweep rate and direction of the B field and current amplitude.

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